This is the second version of the problem set. I'm sorry for neglecting important information in the text of the problems, and a misprint.

Therefore, the Problem 2 is not obligatory. If you will do it, you will receive extra points.

Notations for problems 1 and 2

The Hamiltonian for an atom with magnetic moment $\boldsymbol{\mu}$ placed in a magnetic field (assumed classical) $\boldsymbol{B}$ is

$$\hat{H} = -\hat{\boldsymbol{\mu}}\boldsymbol{B}. \tag{1}$$

Assume, that the magnetic moment of the atom in question is proportional to the vector of Pauli matrices:

$$\hat{\boldsymbol{\mu}} = \gamma \left[\hat{\sigma}_x,\, \hat{\sigma}_y,\, \hat{\sigma}_z\right] \tag{2}$$

Let's assume that the space of states of the magnetic moment is spanned by two vectors, $|+\rangle$ and $|-\rangle$ obeying

$$\hat{\sigma}_z|-\rangle = -|-\rangle \qquad \text{and} \qquad \hat{\sigma}_z|+\rangle = |+\rangle, \tag{3}$$

and therefore at any instant of time t, a state of the system can be written in the form

$$|\psi(t)\rangle = \alpha_(t)|+\rangle + \beta(t)|-\rangle. \tag{4}$$

Problem 1: Motion in a fixed magnetic field

Let's assume that the magnetic field is fixed, and polarized along Z:

$$\boldsymbol{B} = \left[0,\, 0,\, B_0\right]^T. \tag{5}$$

Compute the mean values $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$ averaged in the state of the system at any instant of time t as functions of $\alpha(t = 0)$, $\beta(t = 0)$ and B_0.

Problem 2: Superposition of a fixed magnetic field and an oscillating magnetic field

Let's assume that the magnetic field is constant in the Z direction, but oscillating in the X direction.

$$\boldsymbol{B} = \left[B_1 \cos(\omega t),\, 0,\, B_0\right] \tag{6}$$

a) Use the Schrodinger equation $i\hbar\frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$ to write down equations for the coefficients α and β.

b) Define new variables, $\tilde{\alpha} := e^{i\lambda_1 t}\alpha$ and $\tilde{\beta} := e^{-i\lambda_2 t}\beta$.

Are there such positive values of λ_1 and λ_2 for which the variables $\tilde{\alpha}$ and $\tilde{\beta}$ obey the equation:

$$i\hbar\frac{d}{dt}\begin{bmatrix}\tilde{\alpha}\\ \tilde{\beta}\end{bmatrix} = \tilde{H}\begin{bmatrix}\tilde{\alpha}\\ \tilde{\beta}\end{bmatrix} \tag{7}$$

for time-independent Hamiltonian $\tilde{H}$, neglecting rapidly oscillating terms $e^{i(\lambda_1 t+\lambda_2+\omega)t}$.
?

d) Give explicit form of $\alpha(t)$ and $\beta(t)$ assuming $\alpha(t=0)=1$.

e) What condition has to be meet, to flip the spin with such magnetic fields, i.e. what are criteria to obtain the state $|-\rangle$ at some instant of time?