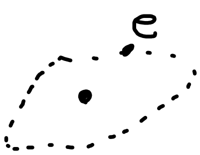
Introduction to atomic physics

Movie 4a

Summary



Spectrum

$$E_{n,l} = -\frac{|E_{GS}|}{n^2} + \Delta E_{\vec{L}\cdot\vec{S}}$$

Role of mass

Hamiltonian for proton and electron:

$$\hat{H} = \hat{H}_{CM} + \hat{H}_{rel}$$

center of mass

$$\hat{H}_{rel} = -\frac{\hbar^2}{2\mu}\Delta - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Hamiltonian as before BUT with $\mu = \frac{m_e \cdot m_{nucleus}}{m_e + m_{nucleus}}$

Harold Urey

Consequences:

$$-\frac{|E_{GS}|}{n^2} \mapsto -\frac{|E_{GS}|}{n^2}\cdot\left(1 + \frac{m_e}{m_{nucleus}}\right)$$

1931, Urey, experiment: "splitting" of spectrum

H $\overset{I}{\rightsquigarrow}$ in agreement with correction $\left(1 + \frac{m_e}{m_{proton}}\right)$

$\overset{I_1}{\rightsquigarrow}$ - in agreement with $\left(1 + \frac{m_e}{2\, m_{proton}}\right)$

DISCOVERY OF DEUTERIUM!! $\leftarrow$

Urey experiment : confirmation that Hydrogen exist in more than one form

spectroscopy $\Rightarrow$ mass of cores of isotopes

1932 - discovery of neutrons (Chadwick)

Muonic atoms

Length scale of atom (radius of Bohr orbit)

$$r_n \propto \frac{4\pi\epsilon_0 \hbar^2}{e^2 m} n^2$$

m ← reduced mass

if m increases → electron closer to core

For different isotopes → small correction

BUT : One can replace electron with muon!!

Muon, μ → charge = charge of electron
→ mass: 200 havier than the mass of electron

sorry for notation. Here μ is muon not mass.

Creation: accelerators

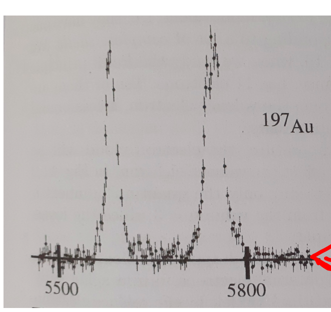
$$\underset{\text{neutron}}{n} + \underset{\text{proton}}{p} = p + p + p + \pi^-$$

π^- ← meson π

$$\pi^- \xrightarrow{\text{slow decay}} \mu + \nu$$

ν ← neutrino

core = neutrons + protons



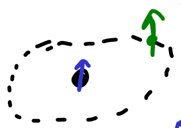
μ INSIDE the nucleus, for spherical core, Observed spectrum, like of HARMONIC OSCILLATOR.

Introduction to atomic physics

Movie 4b

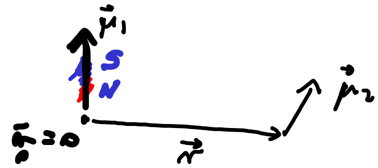
Interaction between magnetic dipoles

"Classical electrodynamics" John David Jackson, Chapters 5.7 (interaction) and 5.8 (hyperfine splitting)



$\vec{\mu}_e = -\frac{e}{2m_e}\gamma \vec{S}_e$, $\gamma \approx 2+\epsilon$

$\vec{\mu}_p = \frac{e}{m_p}\gamma_p \vec{S}_p$



Fermi correction

$$\Delta H_{HFS} = \underbrace{\frac{\mu_0}{4\pi}\frac{1}{\hat{r}^3}\left(\hat{\vec{\mu}}_1\cdot\hat{\vec{\mu}}_2 - 3\left(\hat{\vec{\mu}}_1\cdot\frac{\hat{\vec{r}}}{r}\right)\cdot\left(\hat{\vec{\mu}}_2\cdot\frac{\hat{\vec{r}}}{r}\right)\right)}_{\Delta H^{(1)}_{HFS}} - \underbrace{\frac{2}{3}\mu_0\,\delta(\vec{r})\,\hat{\vec{\mu}}_1\cdot\hat{\vec{\mu}}_1}_{\Delta H^{(2)}_{HFS}}$$

→ atomic clock
→ astrophysics
→ NMR → chemistry, → medicine
⋮

Hyperfine splitting

Enrico Fermi

$$E_{n,l,s} = -\frac{|E_{GS}|}{n^2} + \Delta E_{LS} + \Delta E_{HFS}$$

Assumptions
→ ΔE_{HFS} small
→ external magnetic field – absent $B=0$

$$\Delta E_{HFS} = \int d^3r\, |\psi_{nlms}|^2 \cdot \Delta \hat{H}_{HFS}$$

$$l = 0 \Rightarrow m = 0 \Rightarrow \psi_{n\, l=0\, m=0\, s}(\vec{r}) = F_s(r)$$

$$\int d^3r\, |F(r)|^2 \Delta \hat{H}^{(1)}_{HFS} = 0$$

$$\Delta E_{HFS} = \langle \Delta \hat{H}^{(2)}_{HFS} \rangle = -|F_s(0)|^2 \cdot \frac{2}{3}\mu_0 \hbar^2 \langle \vec{\mu}_e\, \vec{\mu}_p \rangle / \hbar^2$$

$$= \underbrace{+\frac{2}{3}|F_s(0)|^2 \mu_0 \gamma_e \gamma_p \frac{e^2\hbar^2}{2m_p m_e}}_{A} \cdot \left\langle \frac{\vec{S}_e \cdot \vec{S}_p}{\hbar^2} \right\rangle$$

$$\left\langle \frac{\vec{S}_e \cdot \vec{S}_p}{\hbar^2} \right\rangle = \frac{1}{2}\frac{1}{\hbar^2} \left\langle (\vec{S}_e + \vec{S}_p)^2 - \hat{S}_e^2 - \hat{S}_p^2 \right\rangle$$

$$\hat{S}_e^2 = \hbar^2 (s+1)\cdot s \overset{s=\frac{1}{2}}{=} \frac{3}{4}\hbar^2$$

$$\left\langle \left(\hat{\vec{S}}_e + \hat{\vec{S}}_p\right)^2 \right\rangle \overset{\hbar^2 S(S+1)}{=} \begin{cases} 0 \\ 2\hbar^2 \end{cases}$$

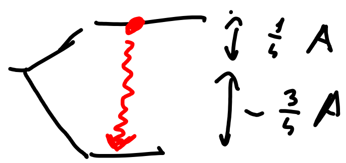
$S = 0$, singlet $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

$S = 1$ triplet $\to$ 3 state

$$\Delta E_{HFS} = A \cdot \begin{cases} -\frac{3}{4} & \text{singlet} \\ +\frac{1}{4} & \text{triplet} \end{cases}$$

GS of hydroge

-13.6 eV



$\updownarrow \frac{1}{4} A$

$\updownarrow -\frac{3}{4} A$

$\updownarrow \hbar\omega$

$\omega = 2\pi \cdot 1.4$ GHz

$\Downarrow$

$\lambda \approx 21$ cm

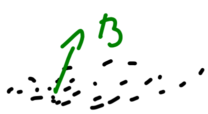
21 cm

21 cm

Lifetime $+\frac{1}{4}A$, $T \approx 10^7$ years

Astrophysics

it happens

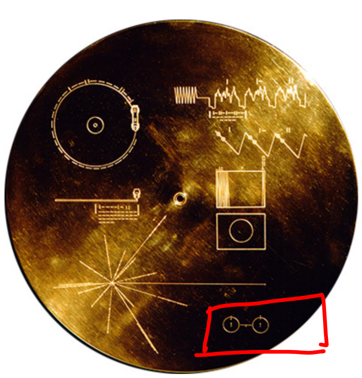


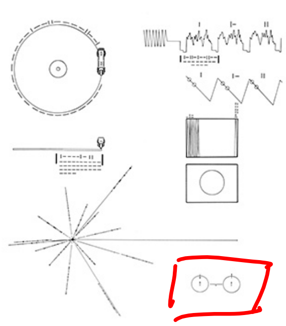
21 cm

$\to$ velocity of atoms in clouds

hydrogen

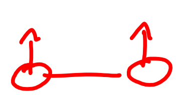
100 K





THE plate
on the board
Pioneer 10 space probe

Where is it now?



21 cm