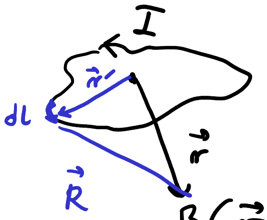
Introduction to Atomic Physics

Lecture 3a

Classical theory - magnetic moment vs angular momentum

I



Biot-Savart

$$\vec A(\vec r) \propto \int \frac{d\vec l}{\sqrt{r^2+r'^2-2rr'\cos\theta}}$$

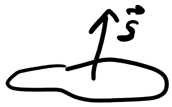
$$B(\vec r) = \nabla \times \vec A$$

$$|\vec r'| << |\vec r|$$

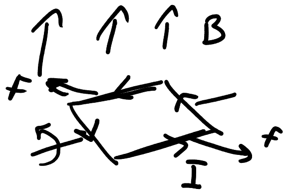
$$\frac{r'}{r} << 1$$

$$\vec A \propto \underbrace{\oint \frac{d\vec l}{r}}_{=0} + \int \frac{d\vec l}{r} \underbrace{\sum_{n=1}^{\infty} \left(\frac{r'}{r}\right)^n \cdot P_n(\cos\theta)}_{\text{dipole moment}}$$

magnetic moment $\longrightarrow$ $\vec\mu = I\vec S$



II



$$\vec F = 0$$

torque $\vec T = \vec\mu \times \vec B$

$$W = -\vec\mu \cdot \vec B$$

• $\boxed{\vec F = -\nabla W = \sum_{i=x,y,z} \mu_i \nabla B_i}$

$$I = \frac{\Delta Q}{\Delta T} = \frac{-e}{\underbrace{2\pi r}_{v}} = \frac{ev}{2\pi r}$$

• $\vec\mu = I\vec S = \frac{ev}{2\pi r}\cdot \pi r^2 \hat s$

$$= \frac{evr}{2}\hat s$$

orbital momentum $\vec L = \vec r \times \vec p$.

$$|\vec L| = (rv)\,m$$

$$\vec\mu = \gamma \vec L$$

$\boxed{\gamma = \frac{e}{2m}}$ ← gyromagnetic

Quantum mechanics

Correspondence

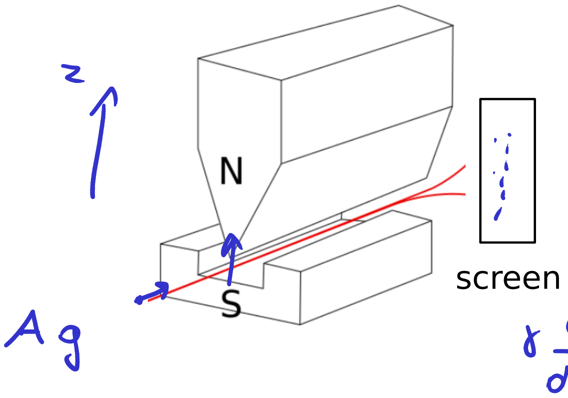
$$\hat{\vec\mu} = \gamma \hat{\vec L}$$

$$\hat{\vec\mu}\,|\mu\rangle = \mu\,|\mu\rangle$$

$\mu = \hbar\gamma m$ m integer $l \in \mathbb{N}$

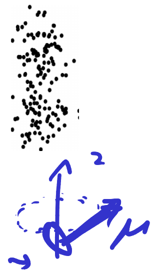
$$-l \le m \le l$$

Stern-Gerlach experiment



$$\nabla B_x = \nabla B_y = 0$$
$$\nabla B_z \neq 0$$
$$F \propto \mu_z \nabla B_z$$

classical expectation

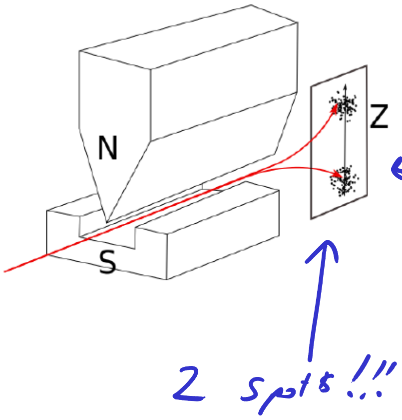


$$\gamma \frac{d\vec{L}}{dt} = \vec{T}$$
$$\frac{d\vec{\mu}}{dt} = \vec{T} = \vec{\mu} \times \vec{B}$$

Stern Gerlach
1921

$$\mu_z = \pm \mu$$

result



$$-\ell\mu_0 \leq \mu_z \leq \ell\mu_0$$

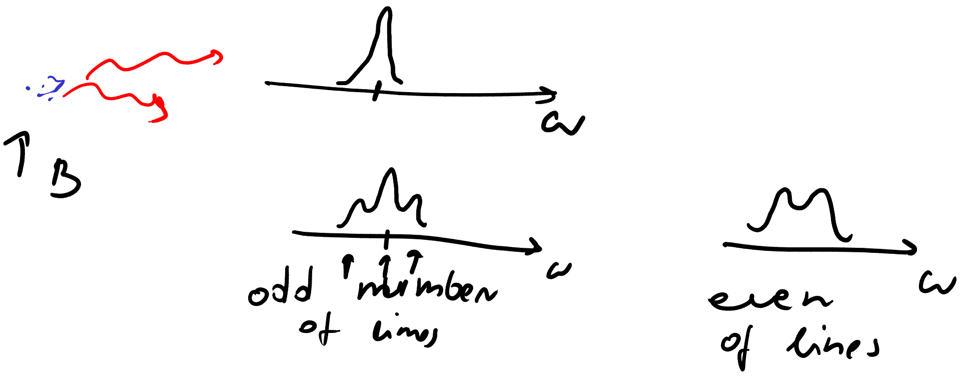
$\ell = 1 \rightarrow$ 3 spots

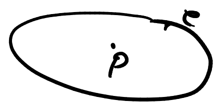
$m = -1, 0, 1$

$\ell = 0 \quad m = 0$

odd number of spots

The other puzzle - the Zeeman effect





$\rightarrow \ell =$ integer

Introduction to Atomic Physics

Lecture 3b

Need for .. spin

$$\hat{\vec{\mu}} = \gamma \hat{\vec{L}} \Longrightarrow \mu_z = \gamma \hbar m$$

$$-l \leq m \leq l$$

$$\hat{L}^2 |\psi_{lm}\rangle = \hbar\, l(l+1) |\psi_{lm}\rangle$$

I. Stern-Gerlach $\Rightarrow$ μ_z - discrete $\Rightarrow$ 2 projections

if l integer $\Rightarrow$ $\mu_z = \underbrace{-l\hbar, \dots, l\hbar}_{\text{odd number}}$

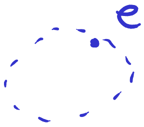
II. Zeeman effect

↑ B

// Sodium: even number of lines

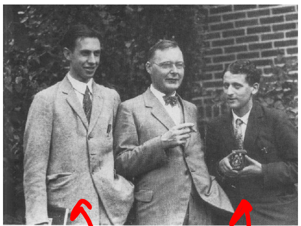
III. Mendelejev periodic table

Pauli $\Rightarrow$ 4 quantum numbers for electron



e: l, m, n',

↑ radial part



Wikipedia

George Uhlenbeck

Samuel Goudsmit

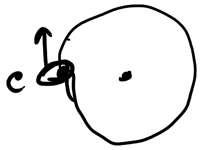
Alsos mission, www.atomicheritge.org

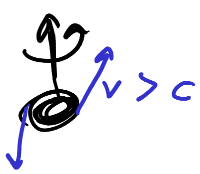
l - half integer

2 definitions

1) $\hat{\vec{L}} = \hat{\vec{r}} \times \hat{\vec{p}} \Rightarrow$ $L_z \propto \frac{\partial}{\partial \varphi}$, $e^{im\varphi}$, $m \in \mathbb{Z}$

2) $\hat{L} \times \hat{L} = i\hbar \hat{L} \Rightarrow l$ integer, half integer





$v > c$ $\Rightarrow$ spin is pure quantum $(l = \frac{1}{2})$

2 dimensional Hilbert space

$$|\uparrow\rangle, \quad |\downarrow\rangle$$

$$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle \quad \Rightarrow \quad |\psi\rangle = \cos\tfrac{\theta}{2}|\uparrow\rangle + e^{i\varphi}\sin\tfrac{\theta}{2}|\downarrow\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$e^{i\alpha}|\psi\rangle \text{ " = " } |\psi\rangle$$

Bloch sphere

Pauli matrices

- $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
- $\hat{S}_x = \frac{\hbar}{2}\hat{\sigma}_x \qquad \hat{S}_y = \frac{\hbar}{2}\hat{\sigma}_y \qquad \hat{S}_z = \frac{\hbar}{2}\hat{\sigma}_z$

$$n_x\hat{S}_x + n_y\hat{S}_y + n_z\hat{S}_z = \vec{n}\cdot\vec{S} \qquad \vec{n}\cdot\vec{n} = 1$$

$$e^{+i\alpha\,\vec{n}\vec{S}/\hbar} \longleftarrow \text{rotation in the space of states}$$

Ex. $\vec{n} = [0, 0, 1]$

$$e^{+i\alpha\,\vec{n}\vec{S}/\hbar} = e^{+i\alpha\,\hat{S}_z/\hbar} = e^{i\alpha\,\hat{\sigma}_z/2}$$

$$e^{i\alpha\sigma_z/2}\left(\cos\tfrac{\theta}{2}|\uparrow\rangle + e^{-i\varphi}\sin\tfrac{\theta}{2}|\downarrow\rangle\right) =$$

$$= e^{i\frac{\alpha}{2}}\left(\cos\tfrac{\theta}{2}|\uparrow\rangle + e^{-i(\varphi+\alpha)}\sin\tfrac{\theta}{2}|\downarrow\rangle\right)$$

Strange things $\quad \langle\psi|\xi\rangle = 0$

$$\alpha = 2\pi \qquad e^{i\alpha\sigma_z/2} = -\mathbb{I}$$

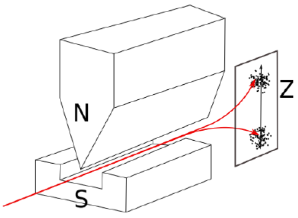
All rotation $\mathbb{R}^3$

$$SO(3)$$

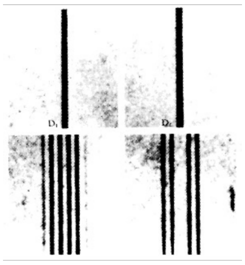
$$e^{i\alpha\vec{n}\vec{S}} \longrightarrow SU(2)$$

Introduction to Atomic Physics

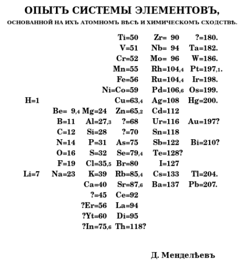
Lecture 3c



Stern-Gerlach experiment (1922)



anomalous Zeeman effect (1896)



ОПЫТЪ СИСТЕМЫ ЭЛЕМЕНТОВЪ,

ОСНОВАННОЙ НА ИХЪ АТОМНОМЪ ВѢСѢ И ХИМИЧЕСКОМЪ СХОДСТВѢ.

			Ti=50	Zr=90	?=180.
			V=51	Nb=94	Ta=182.
			Cr=52	Mo=96	W=186.
			Mn=55	Rh=104,4	Pt=197,1.
			Fe=56	Ru=104,4	Ir=198.
			Ni=Co=59	Pd=106,6	Os=199.
H=1			Cu=63,4	Ag=108	Hg=200.
	Be=9,4	Mg=24	Zn=65,2	Cd=112	
	B=11	Al=27,3	?=68	Ur=116	Au=197?
	C=12	Si=28	?=70	Sn=118	
	N=14	P=31	As=75	Sb=122	Bi=210?
	O=16	S=32	Se=79,4	Te=128?	
	F=19	Cl=35,5	Br=80	I=127	
Li=7	Na=23	K=39	Rb=85,4	Cs=133	Tl=204.
		Ca=40	Sr=87,6	Ba=137	Pb=207.
		?=45	Ce=92		
		?Er=56	La=94		
		?Yt=60	Di=95		
		?In=75,6	Th=118?		

Д. Менделѣевъ

Mendeleev table

SPIN

$$\hat{\vec{L}} \times \hat{\vec{L}} = i\hbar \hat{\vec{L}}$$

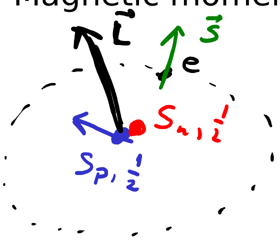
$$L^2 |\psi\rangle = \hbar^2 \ell(\ell+1) |\psi\rangle$$

$$L_z |\psi\rangle = \hbar m |\psi\rangle$$

$$-\ell \le m \le \ell$$

$$\begin{cases} \vec{S}^2 |\psi\rangle = \frac{3}{4}\hbar^2 |\psi\rangle \\ m = \pm\frac{1}{2} \end{cases}$$

Magnetic moments due to spin



$$\left.\begin{aligned} \hat{\vec{L}} &= \hat{\vec{r}} \times \hat{\vec{p}} \\ \hat{\mu}_L &= -\frac{e}{2m_e} \hat{\vec{L}} \end{aligned}\right\}$$

$$\vec{\mu}_s = g_s \left(-\frac{e}{2m_e}\right) \vec{S}$$

gyromagnetic factor

THEO, EXPERIMENT gs = 2(1 + 0.001 159 6522)

$$\hat{\vec{\mu}}_p \approx 2.79 \frac{e}{m_p} \hat{\vec{S}}_p$$

$$m_p \gg m_e$$
$$m_p \simeq 1836\, m_e$$

$$\hat{\vec{\mu}}_n \approx -1.9 \frac{e}{m_n} \vec{S}_n$$!!?

puzzle : solution quark model of matter

Fine and hyperfine structure

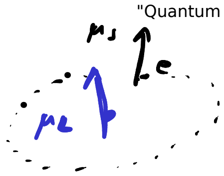
$E_n = -\frac{E}{n^2}$ + corrections

$\vec{\mu}_L \longleftrightarrow \vec{\mu}_S$ => fine structure

$\vec{\mu}_p, \vec{\mu}_n \longleftrightarrow \begin{matrix}\vec{\mu}_L \\ \mu_s\end{matrix}$ => hyperfine structure

Fine structure: LS coupling

More details:
V B Berestetskii, L. P. Pitaevskii, E.M. Lifshitz
"Quantum Electrodynamics" Chapter: "Particles in an external field"



$|\vec{\mu}_L|(1T)\mu_s$

1) Coordinate system, with static electron

$$\vec{E} = \frac{e}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3} \xrightarrow[\text{frame}]{\text{moving}} \vec{B} = -\frac{\vec{v}\times\vec{E}}{c^2}$$

$$\boxed{\vec{B}} = \frac{e}{4\pi\varepsilon_0 c^2}\frac{1}{r^3}\frac{1}{m}\overbrace{(-m\vec{v})\times\vec{r}}^{\vec{L}}$$

$$= \frac{e}{4\pi\varepsilon_0 c^2}\frac{1}{mr^3}\vec{L}$$

2) Laboratory frame $\longrightarrow \frac{1}{2}$

Energy $\to \vec{\mu}_s$ in $\vec{B}$ => $-\vec{\mu}_s\vec{B}$

$$\Delta\hat{E} \overset{(x)}{=} \frac{e}{4\pi\varepsilon_0 c^2}\frac{1}{m}\frac{1}{\hat{r}^3}\frac{e}{2m_e}\hat{\vec{L}}\cdot\hat{\vec{S}}$$

$$(x)\ \frac{e^2}{4\pi\varepsilon_0 m^2c^2}\frac{1}{\hat{r}^3}\hat{L}\cdot\hat{S}$$

$$\hat{\vec{L}}\cdot\hat{\vec{S}} = \frac{1}{2}\left((\vec{L}+\vec{S})^2 - \vec{L}^2 - \vec{S}^2\right) =$$

$$\frac{1}{2}\hbar^2\left[j(j+1) - l(l+1) - s(s+1)\right]$$

$j = l \pm \frac{1}{2}$

$+\frac{1}{2}$ ...... $\uparrow_L \uparrow_S$

$-\frac{1}{2}$ $\uparrow_L \downarrow_S$

$$\Delta\hat{E} \overset{(x)}{=} \underbrace{\frac{\hbar^2 e^2}{4\pi\varepsilon_0 m^2c^2}}_{\alpha^2 E_H}\frac{1}{\hat{r}^3}\frac{\vec{L}\cdot\vec{S}}{\hbar^2}$$

$$\alpha^2 E_H \cdot \left(\frac{a}{\hat{r}}\right)^3\left(\frac{\vec{L}\cdot\vec{S}}{\hbar^2}\right)$$

13.6eV

$$\alpha = \frac{e}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$$

$$\left\langle\left(\frac{a}{\hat{r}}\right)^3\right\rangle \propto \frac{Z^4}{n^2 l^2}$$

$Z \to 1$ hydrogen