13/01/2021

Last time:

→ Zeeman effect.

- strong Zeeman:

$$E^{(1)} \sim B(m_\ell + 2m_s)$$

- weak Zeeman:

$$E^{(1)} \sim B\, m_j\, g(\ell)$$

B - oscillates in time

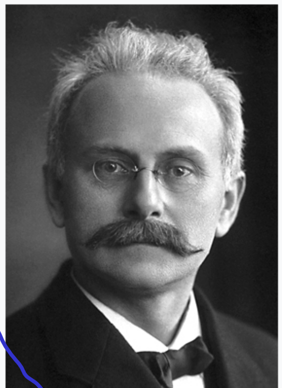
movies 5 a)

Today's lecture: Stark effect

an atom in electric field

Stark effect in 1913

Johannes Stark



Antonino Lo Surdo

Stark - Lo Surdo effect

Static electric field (DC)

1) Quadratic Stark effect

non-degenerate case

2) linear Stark effect

degenerate case

• oscillating electric field (AC)

electric dipole moment $d = \alpha E$

$$V_{int} = \frac{1}{2} d E \sim \frac{1}{2} \alpha E^2 \sim E^2$$

Remind perturbation theory:

$$H = H_0 + \lambda H' \qquad H_0 |n^0\rangle = E_n^{(0)} |n^0\rangle$$

first order correction:

$$E_n^{(1)} = \langle n^0 | H' | n^0 \rangle$$

$$|n^1\rangle = \sum_{m\neq n} \frac{|m^0\rangle\langle m^0|H'|n^0\rangle}{E_n^{(0)} - E_m^{(0)}}$$

Second order correction:

$$E_n^{(2)} = \sum_{m\neq n} \frac{|\langle m^0|H'|n^0\rangle|^2}{E_n^{(0)} - E_m^{(0)}}$$

Hydrogen atom

$H = H_0 + H'$

$H_0 = H_{kin} + H_{cou}$ $\qquad H_0|n^0\rangle = E_n^{(0)}|n^0\rangle$

$H' = H_{stark} = -\vec{d}\,\vec{\mathcal{E}}$ $\qquad \vec{\mathcal{E}} = (0, 0, \mathcal{E})$

$= -e z \mathcal{E}$ $\qquad \vec{d} = -e|\vec{x}|$

First order correction:

$E_n^{(1)} = \langle n^0 | H' | n^0\rangle$

$\sim \int d^3r\, \psi^*_{nlm}\, z\, \psi_{nlm}$

$|\ \ |^2 \Rightarrow$ even parity

$z \rightarrow$ odd parity

$E_n^{(1)} = 0$

Second order:

$$E_n = E_n^{(0)} + \sum_{n \neq m} \frac{|\langle n^0 | H' | m^0 \rangle|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\Delta E_n \sim E_n^{(0)} \sim e^2 \mathcal{E}^2 \sum_{m \neq n} \frac{|\langle n^0 | z | m^0 \rangle|^2}{E_n^{(0)} - E_m^{(0)}}$$

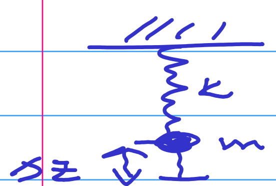
- Expectation values for:

$\langle H' \rangle = ?$ A A) $-\langle \vec{d} \rangle \mathcal{E}$

$\langle H_0 + H' \rangle = ?$ A/B B) $-\frac{1}{2} \langle \vec{d} \rangle \mathcal{E}$

https://ocw.mit.edu/courses/physics/8-421-atomic-and-optical-physics-i-spring-2014/video-lectures/le

Based on Kellerle idea:



↓ gravity $\Delta E = -mg\,\overline{\Delta z}$

total: $-\frac{1}{2} \Delta E$

$\langle H' \rangle = A = -\langle d \rangle \mathcal{E}$

$\langle H_0 + H' \rangle = -\frac{1}{2} \langle d \rangle \mathcal{E}$

1) When the above works? for s states $(l = 0)$

2) what with spin degrees of freed?

Linear Stark effect.

$n = 2$

ν	n	ℓ	m
1	2	0	0
2	2	1	0
3	2	1	-1
4	2	1	1

$$\phi(r) = \sum_\nu c_\nu \psi_\nu$$

$$\psi_\nu \equiv \psi_{n\ell m}$$

$$H_{\mu\nu} = \int d^3r\, \psi_\mu^* H \psi_\nu$$

$$= E_\nu + \int d^3r\, \psi_\mu^* H' \psi_\nu$$

$$\begin{pmatrix} H_{11} & H_{12} & \cdots \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = E \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$$

diagonal term $= 0$

10^{00} $\langle 200 | H' | 211 \rangle = ? = 0$

10^{10} $\langle 200 | H' | 21-1 \rangle = ? = 0$

$$\boxed{\langle 200 | H' | 210 \rangle \neq 0 = Y}$$

$$\int d^3r\, \psi_\mu^* z\, \psi_\nu = \langle \mu | z | \nu \rangle$$

$$\int_0^\infty x^n e^{-an}\,dx = \frac{n!}{a^{n+1}}$$

n	l	m	$\psi_{n,l,m}(r,\theta,\phi)$
1	0	0	$\frac{1}{\sqrt{\pi}a_0^{3/2}}e^{-r/a_0}$
2	0	0	$\frac{1}{4\sqrt{2\pi}a_0^{3/2}}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$
2	1	0	$\frac{1}{4\sqrt{2\pi}a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\cos\theta$
2	1	±1	$\frac{1}{8\sqrt{3\pi}a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\sin\theta\, e^{\pm i\phi}$
3	0	0	$\frac{1}{81\sqrt{3\pi}a_0^{3/2}}\left(27-18\frac{r}{a_0}+2\frac{r^2}{a_o^2}\right)e^{-r/2a_o}$
3	1	0	$\frac{1}{81\sqrt{3\pi}a_0^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}\cos\theta$
3	1	±1	$\frac{1}{81\sqrt{3\pi}a_0^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}\sin\theta\, e^{\pm i\phi}$
3	2	0	$\frac{1}{81\sqrt{6\pi}a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}\left(3\cos^2\theta-1\right)$
3	2	±1	$\frac{1}{81\sqrt{\pi}a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}\sin\theta\cos\theta\, e^{\pm i\phi}$
3	2	±2	$\frac{1}{162\sqrt{\pi}a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}\sin^2\theta\, e^{\pm 2i\phi}$

$$\begin{pmatrix} E^{(0)}_{n=2}-E & \gamma & 0 & 0 \\ \gamma & E^{(0)}_{n=2}-E & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = 0$$

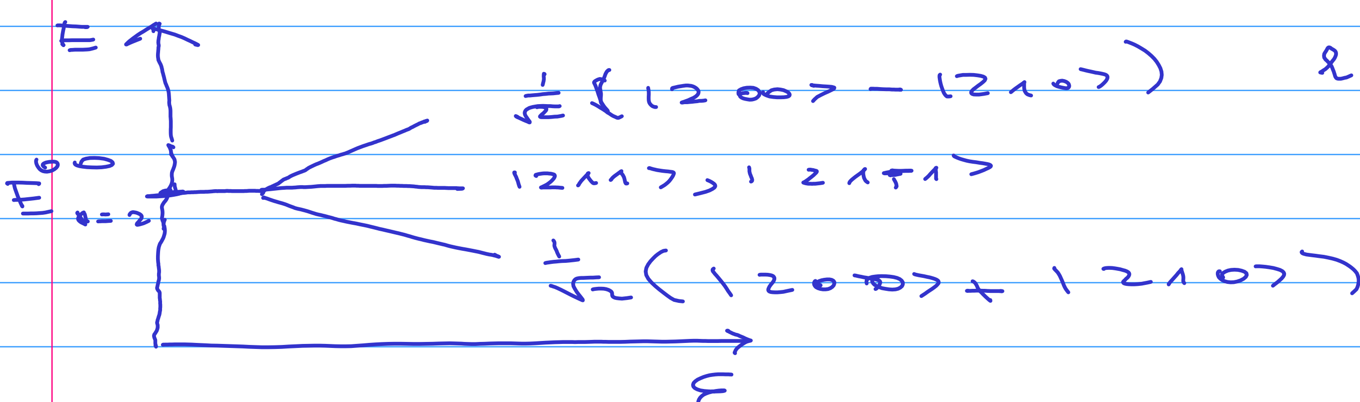
$$\phi_4 = \psi_4 = \psi_{21-1} \qquad E = E^{(0)}_{n=2}$$

$$\phi_3 = \psi_3 = \psi_{211} \qquad E = E^{(0)}_{n=2}$$

$$\phi_2 = \frac{1}{\sqrt{2}}\left(\psi_{200}+\psi_{210}\right), \quad E = E^{(0)}_{n=2}+\gamma$$

$$\phi_1 = \frac{1}{\sqrt{2}}\left(\psi_{200}-\psi_{210}\right), \quad E = E^{(0)}_{n=2}-\gamma$$

$$\gamma = -3ea_0\mathcal{E}$$



$l \neq l'$

Summary:

Zeeman	Stark
strong	degenerate
$E^{(1)} \sim B(m_\ell + 2m_s)$	(linear)
	$E^{(1)} \sim \mathcal{E} \begin{cases} E^{(0)}_{n=2} \\ E^{(0)}_{n=2} \pm \gamma \end{cases}$
weak	quadratic
$E^{(1)} \sim B\, g\, m_j$	$E^{(1)} \sim \mathcal{E}^2 \mathcal{L}$

27th of January