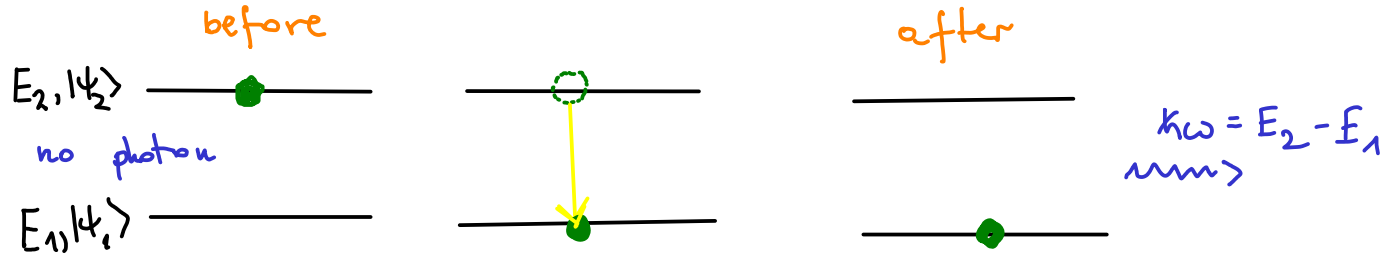
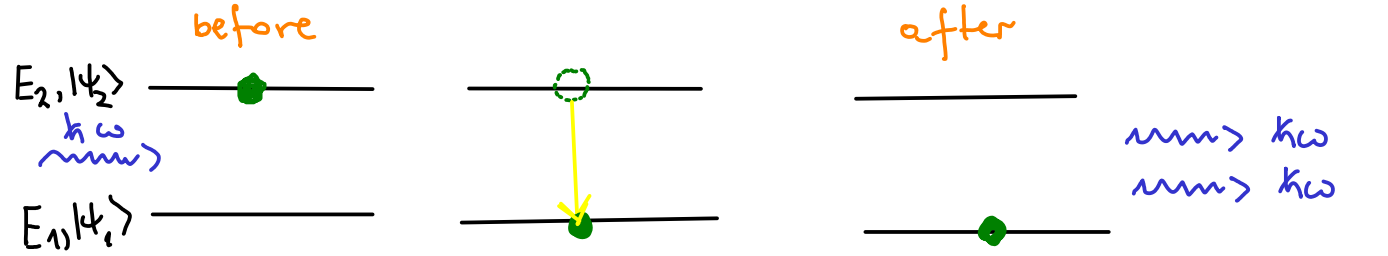
Emission and absorption of electromagnetic radiation by atoms

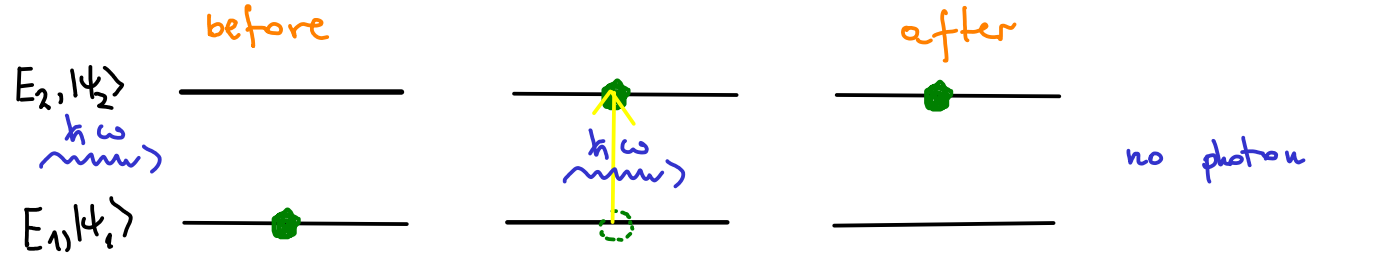
1) spontaneous emission



2) stimulated emission



3) absorption



RATE EQUATIONS

N_1 in the ground state & N_2 atoms in $|\psi_2\rangle$

note: A_{21}

$$\frac{dN_2}{dt} = -A_{21} N_2 \qquad N_2(t) = N_2(0)\, e^{-t/\tau}$$

$$\tau = \frac{1}{A_{21}}$$

the total rate for the ground state to be excited

is $\rho(\omega_0) B_{12}$ $\qquad \omega \approx \omega_0 \equiv \frac{E_2 - E_1}{\hbar}$

stimulated emition $\rho(\omega_0) B_{21}$

$$\left[\begin{aligned} \frac{dN_2}{dt} &= \rho(\omega_0)\left(B_{12}N_1 - B_{21}N_2\right) - A_{21}N_2 \\ \frac{dN_1}{dt} &= -\rho(\omega_0)\left(B_{12}N_1 - B_{21}N_2\right) + A_{21}N_2 \end{aligned}\right] \text{ Rate equations}$$

A_{21}, B_{12}, B_{21} → Einstein's coefficients

$$\rho(\omega_0) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$|4_1\rangle$, g_1 - degeneracy, E_1

$|4_2\rangle$, g_2 - degeneracy, E_2

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\hbar\omega_0/k_B T}$$

$$\rho(\omega) = \frac{\hbar}{\pi^3 c^3} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}$$

$$\rho(\omega_0) = \frac{A_{21}}{B_{12}\left(\frac{g_1}{g_2}\right) e^{\hbar\omega_0/k_B T} - B_{21}}$$

$$g_1 B_{12} = g_2 B_{21}$$

$$A_{21} = \frac{\hbar \omega_0^3}{\pi^2 c^3} B_{21}$$

$\delta \gg \Omega \Rightarrow |\omega - \omega_0| \gg \Omega$

$$P_2(t) \approx \frac{\Omega^2}{\delta^2} \sin^2\left(\frac{\delta t}{2}\right)$$

$$\hbar \Omega = e \mathcal{E} \langle \psi_1 | z | \psi_2 \rangle$$

$$P_2(t) = \frac{e^2 \pi}{\varepsilon_0 \hbar^2} \rho(\omega_0) |\langle \psi_1 | z | \psi_2 \rangle|^2 t$$

Rate for absorption $= \dot{P}_2(t) =$

$$B_{12} = \frac{e^2 \pi}{\varepsilon_0 \hbar^2} \underbrace{|\langle \psi_1 | z | \psi_2 \rangle|^2}_{\text{m.e.}}$$

$$B_{21} = \frac{g_1}{g_2} B_{12} \Rightarrow A_{21} = \frac{\hbar \omega_0^3}{\pi^3 c^2} B_{21}$$

$$\tau = \frac{1}{A_{21}} \sim \frac{1}{|\langle \psi_1 | z | \psi_2 \rangle|^2}$$